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Lateral Pressure of
Granular Substances

Civil Engineering

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LATERAL PRESSURE OF GRANULAR
SUBSTANCES

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BY

POMEROY SINNOCK

THESIS

FOR THE

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THIS IS TO CERTIFY THAT THE THESIS PREPARED UNDER MY SUPERVISION BY

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ENTITLED LATERAL PRESSURE OF GRANULAR SUBSTANCES

IS APPROVED BY ME AS FULFILLING THIS PART OF THE REQUIREMENTS FOR THE

DEGREE OF Bachelor of Science in Civil Engineering

John P. Brooks

Instructor in Charge.

APPROVED:

John P. Brooks

HEAD OF DEPARTMENT OF Civil Engineering

LATERAL PRESSURE OF GRANULAR SUBSTANCES

INTRODUCTION

The object of this paper is to investigate the subject of lateral pressure as exerted in any mass of material having a granular make-up. This investigation will consist in making a synopsis of existing theories and experiments and those which approach most nearly the results derived in actual practice will be taken as the method for determining the amount of this lateral pressure.

Heretofore, most construction involving the question of lateral pressure, such as grain bins, retaining walls and the like have been designed, more or less, by "the rule of thumb" method resulting from much and varied experience along that line. Now-a-days, when economical design is of such great importance, it is desirable that the subject should be treated scientifically. At present, however, the subject cannot be treated as a problem that permits of exact theoretical solution because the existing theories are rather defective due to the fact that they have not had practical verification. If the theories, as they now exist, are treated intelligently-- and this can only be done by those engineers who are well acquainted with the practical side of the subject-- they will place the design of structures involving lateral pressure, upon a scientific basis.

The great bulk of available data consists of mathematical deductions founded on the one theory that the lateral

pressure is caused by a wedge-shaped mass of the material retained, bounded on the one side by the back, the supporting wall, and on the other by the plane of rupture or plane of least resistance, which plane makes some certain angle with the horizontal greater than the angle of repose of the material in question. Some, perhaps most of the formulas thus deduced, are both deceiving and erroneous because they disregard the practical side of the problem and make assumptions which are not true; also most of the deductions are made without experimental variation. (This condition is more so with earth masses than with other granular masses.)

The lateral pressure exerted in a mass of grain, such as wheat, corn, barley, etc., is more definitely determined than in the case of earth, and different experimenter's results are more concordant throughout. This is due to the fact that these materials, grains, have a consistent make-up, each individual particle being nearly symmetrical with all the others, and separated from them; this does away with cohesive action which is such a strong factor effecting the action of lateral pressure in an earth mass.

There are many different kinds of earth and, perhaps, no two kinds will act the same, consequently there are a great many conditions to deal with. However, classifications can be made which will confine the results to reasonable accuracy.

No attempt will be made by the author of this paper to advance any new theories nor to dictate just how grain bins, retaining walls and other structures involving this

question, shall be designed but it is his purpose to compile, in a systematic way, all of the available and important data on lateral pressure in granular masses.

GRAIN PRESSURE

In 1883, Isaac Roberts carried on tests in a grain bin 6 ft square and 40 ft deep. In this test he ascertained that the greater part of the pressure or grain load was carried by the bin walls, and that both the vertical and lateral pressures were light. Also that the maximum pressure upon the bottom of the bin was reached when the depth of grain in the bin had reached approximately $3\frac{1}{3}$ times the horizontal dimensions of the bin.

In 1895, H. A. Jansen made a number of tests on small square bins with a view of obtaining the proportion of weight of grain in the bin that would rest on the bottom and what part would be carried by the walls. His bins were of approximately the same depth but of varying cross-section. The system used by him consisted in supporting the bin walls upon 4 jack-screws while in the bottom of the bin was placed a loosely fitting board resting upon a platform scale. By filling the bin with grain the proportion of weight resting upon the bottom was recorded on the scale. The bin was then slightly raised, by means of the jack screws and, owing to the friction of the grain on the bin walls, this also relieved part of the bottom pressure and the beam would drop; added weights were then placed in the beam and the filling

continued. Such readings were taken until the bin was full. His tests were thus carried out in four different sized bins, but were to obtain the vertical pressure only as he found that having the proportion of weights resting upon the bottom and carried by the bin sides he could calculate the lateral pressure (having obtained the angle of repose of the grain). From this information he derived a formula, by which the pressure in different sized bins could be calculated.

In 1897 W. Airy carried on a set of experiments which were a valuable addition to the subject at hand. He considered grain as a semi-fluid mass, assuming that the pressure on the sides of the bin is the maximum pressure due to a wedge-shaped mass which may be supposed to separate from the general mass and the angle of slope of the particular wedge-shaped mass, has to be determined. There is friction between bin and grain and also between grain and grain.

He considers two cases viz., when there is a small amount of grain in the bin and the intersection of the planes of parting of the wedge-shaped mass comes without; the second case, when this intersection falls within the mass. However, he proves that this distinction need not be made as the same expression for the vertical and lateral pressures fits both cases. He also proved that P varies as h , the breadth of this bin.

A table was constructed, by him, from experiments carried on in a 10 ft x 10 ft bin holding wheat weighing 50# per cubic foot. (See Table I)

-- TABLE I --

W. Airy's Test Smooth wooden Bin, wheat weighing 50 $\frac{#}{\text{cu}}$ per
cubic foot. Coef. of frict. wheat on wheat = 0.466

Coef. of frict. wheat on sides = 0.361

Depth of grain in Bin Ft.	Value of Tan θ for Max. Press. on Sides	Weight of Grain in Bin lb.	Press. on sides/ft. run of Hor. cir. #s	Weight of grain carried on sides lb.	Pressure in lb per sq.in.	
					Lat.	Ver.
5	1.294	25000	210	21966	0.29	1.53
10	1.294	50000	840	37864	0.87	2.62
15	1.422	75000	1878	47882	1.44	3.32
20	1.708	100000	3169	54340	1.79	3.76
25	1.967	125000	4625	58315	2.02	4.03
30	2.205	150000	6214	60270	2.20	4.18
35	2.427	175000	7900	60924	2.34	4.23
40	2.665	200000	9657	60553	2.44	4.20
45	2.832	225000	11488	59113	2.55	4.10
50	3.019	250000	13386	56706	2.63	3.93
55	3.198	275000	15331	53620	2.68	3.72
60	3.369	300000	17305	50116	2.74	3.49
65	3.535	325000	19333	45846	2.82	3.18
70	3.694	350000	21385	41201	2.85	2.86
75	3.848	375000	23503	35617	2.92	2.48
80	3.997	400000	25617	30091	2.93	2.09
85	4.142	425000	27773	23958	2.94	1.66
90	4.283	450000	29937	17710	3.00	1.23
95	4.420	475000	32119	11302	3.02	0.78
100	4.555	500000	34326	4333	3.13	0.30

Let h = height of grain, b = breadth of bin. Where $\frac{h}{b}$ = constant,
the pressure on the sides and bottom varies as b .

Suppose for example a bin 80 ft deep and 20 ft square.

Here $\frac{h}{b} = 4$. Referring to table $\frac{40}{10} = 4$. Therefore total pressure
on sides of 20 ft bin = $\frac{20}{10} \times 386380 = 3,090,240\frac{\#}{\text{ft}}$

From his conclusions that rectangular bins of the same shape but different sizes and for which $\frac{h}{b}$ = a constant, the pressure on sides and bottom varies as b , the computations of these pressures, both lateral and bottom, can be made upon reference to his tables.

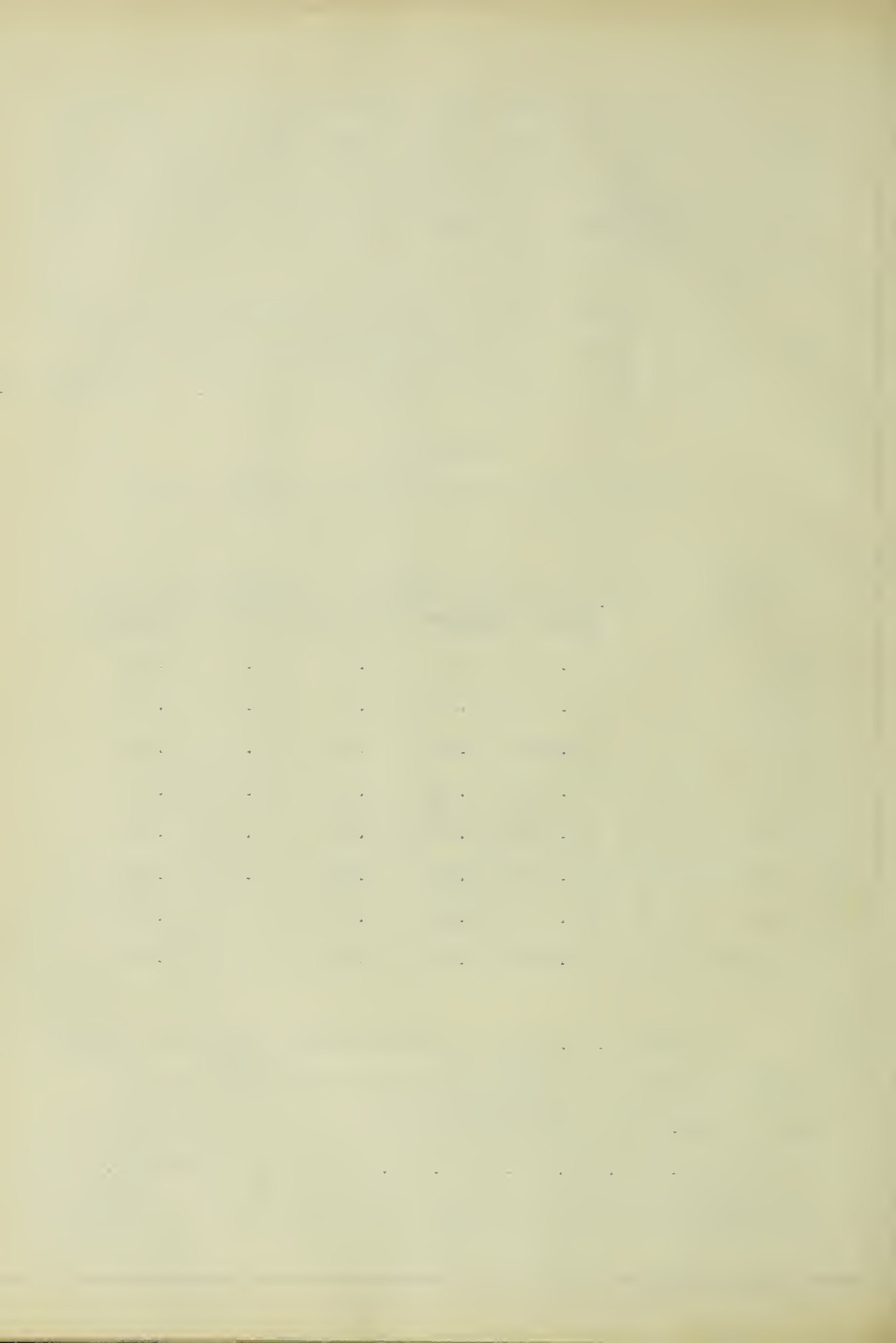
He also gives a table of the coefficients of friction for different grains on different materials, which values correspond to those found by Jamieson and others. See Table II.

TABLE II

-- Coefficients of friction for different grains --

Kind of Grain	Wt per cu. ft.	Coef. of friction				
		Grain on Grain	Grain on Rough Bds	Grain on Sm. Boards	Grain on Iron	Grain on Cement
Wheat	49	0.466	0.412	0.361	0.414	0.444
Barley	39	0.507	0.424	0.325	0.376	0.452
Oats	28	0.532	0.450	0.369	0.412	0.466
Maize	44	0.521	0.344	0.308	0.374	0.423
Beans	46	0.616	0.435	0.322	0.366	0.442
Peas	50	0.472	0.287	0.268	0.263	0.296
Tares	49	0.554	0.434	0.359	0.364	0.394
Linseed	41	0.450	0.407	0.308	0.339	0.414

In 1900, J. A. Jamieson carried out a series of experiments which are the most complete and most satisfactory up-to-date. His tests were carried out in a full-sized bin of the Can. Pac. El. Co. at W. St. Johns, New Brunswick. The dimensions of the bin were 12 ft x 13 ft-6in. in cross

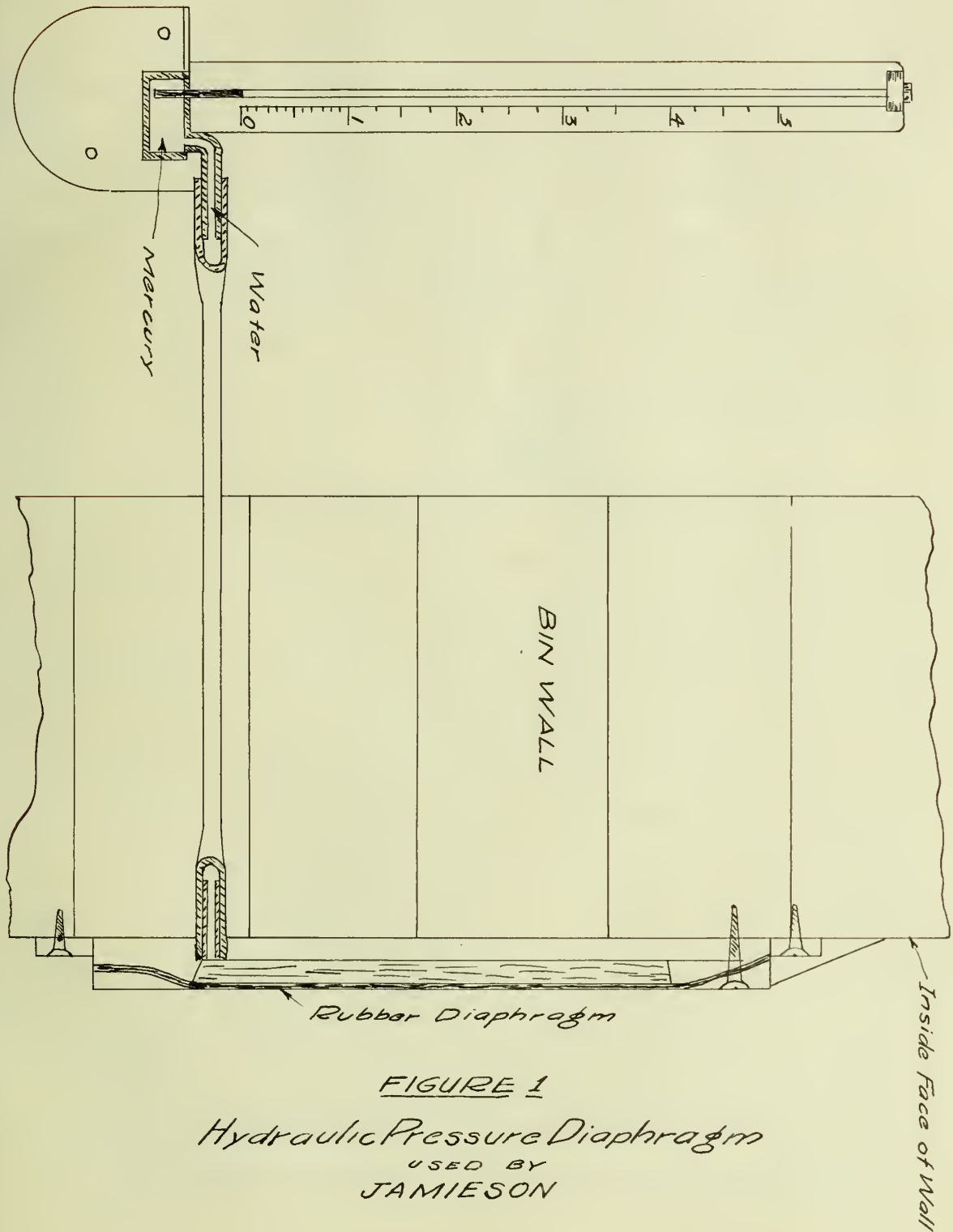


section and 67 ft.- 6 in. in height. The grain used was Manitoba wheat weighing 49.4# per cubic ft. A diaphragm for measuring the lateral pressure was located a short distance above the hopper bottom with faces vertical; and another was located on the hopper bottom with face horizontal to determine the vertical pressure. Figure I illustrates the construction of these diaphragms consisting of the rubber face which transferred the local pressure through the water to the mercury column whose height on the scale read the pressure in pounds per square inch.

The wheat was drafted into the bin in the usual manner, each draft weighing 30000# giving a depth of 3 ft.-9 in. for each added increment. The guages were watched as the wheat was running in and the maximum reading recorded. This method was continued until the bin was full. It was thus allowed to stand for 18 hours, in which time there was no appreciable change in the guage readings.

The grain was now allowed to run off in 30000# increments the maximum guage reading being taken during each as before. The rate of drawing off was 9000 bushels per hour. The pressure readings fluctuated considerably, the maximum increase over filling readings was 4%. The position of the diaphragms was changed and the test repeated without much change in readings.

The pressures, vertical and lateral, were then plotted, the maximum readings of the different tests being used. Curves and tables are here given showing the results of these tests (See tables III and IV). Figure II shows the relation





between the pressure, both lateral and vertical, and the height of grain in the bin. The black graph is this relation provided the grain were a true liquid i.e., water with the same specific gravity as the wheat.

He also carried out experiments in model bins constructed with different inside linings such as corrugated steel, wood both rough and smooth, steel, etc. They were also of different dimensions, some 12" square, others 6" square, while all were 6 ft.- 6 in. in depth.

From the results of these tests he constructed tables of the form shown on preceding page. These results differ but little from those obtained from the experiments carried on in the large bins.

From these tables he draws the following conclusions:

First, assuming that the coefficient of friction within the mass for any given kind of strictly dry granular mass is constant, then there must exist a constant ratio between the horizontal and vertical pressures for all depths of bin. The ratio as determined is 0.6, the angle being 38.

Second, since there is friction between the grain and bin walls, the lateral pressure will cause the wall to act as a column and support part of the weight of the grain. The walls are less compressible than the grain and the greater the pressure-friction the more the walls will support, until the pressure-friction reaches a value equal to the weight of a horizontal layer of the grain, say 1-in. thick, which will give a maximum lateral and vertical pressure.

TABLE III

Jamieson. Bottom Pressure Test. Bin 12 ft. x 13.5 ft

Wheat weighing 49.9# per cubic ft.

Grain weighed into Bin lb.	Height of grain column Ft.	Press. of grain on diaph- ragm lb/sq. in.	Grain carried on Bottom		Grain carried on Sides	
			Weight lb.	% Tot. weight of grain	Weight lb.	% Tot. weight of grain
30000	3.75	1.118	26081	86.9	3919	13.1
60000	7.50	1.948	45443	75.7	14556	24.3
90000	11.25	2.499	58297	64.7	31704	35.3
120000	15.00	2.927	68291	56.9	51719	43.1
150000	18.75	3.427	75746	50.4	74254	49.6
180000	22.50	3.482	81228	45.1	98772	54.9
210000	26.25	3.635	84797	40.3	125203	59.7
240000	30.00	3.752	87527	36.4	152473	63.6
270000	33.75	3.843	89650	33.2	180350	66.8
300000	37.50	3.924	91539	30.5	208461	69.5
330000	41.25	3.987	93009	28.1	236991	71.9
360000	45.00	4.041	94268	26.1	265732	73.9
390000	48.75	4.077	95108	24.3	294892	75.7
420000	52.50	4.095	95528	22.7	324472	77.3
450000	56.25	4.113	95948	21.3	354052	78.7
480000	60.00	4.129	96321	20.1	383679	79.9
510000	63.75	4.129	96321	18.8	413679	81.2
540000	67.50	4.129	96321	17.8	443679	82.2

carried on bottom = 96321 16500 (in hopper) = 112821

" " sides = 443679

Total in bin = 556500 lb.

TABLE IV

Jamieson. Side Pressure Test. Cribbed Wooden Bin 12 ft.
by 13.5 ft.

Grain Weighed into Bin lb.	Height of Grain Col. Ft.	Equivalent Fluid Press. lb/sq. in.	Side Press. of grain on diaph. lb/sq. in.	Side Press per section lb.
30000	3.75	1.386	0.343	9446.22
60000	7.5	2.573	0.938	25832.52
90000	11.25	3.859	1.317	36270.18
120000	15.00	5.145	1.615	44477.10
150000	18.75	6.431	1.804	48682.16
180000	22.50	7.718	2.011	55382.94
210000	26.25	9.004	2.111	58136.94
240000	30.00	10.290	2.201	60615.54
270000	33.75	11.576	2.278	62736.12
300000	37.50	12.863	2.345	63581.30
330000	41.25	14.149	2.381	65672.74
360000	45.00	15.435	2.417	66564.18
390000	48.75	16.721	2.435	67059.90
420000	52.50	18.008	2.453	67555.62
450000	56.25	19.294	2.453	67555.62
480000	60.00	20.580	2.453	67555.62
510000	63.75	21.866	2.462	67803.48
540000	67.50	23.153	2.462	67803.48

Total side pressure = 1,004,631.66 lb.

Vertical pressure = 4.129# per square inch.

Lateral pressure = 2.462# " " "

Vertical pressure = 1.66 times the lateral pressure, or

Lateral pressure = 59.6% of vertical pressure.

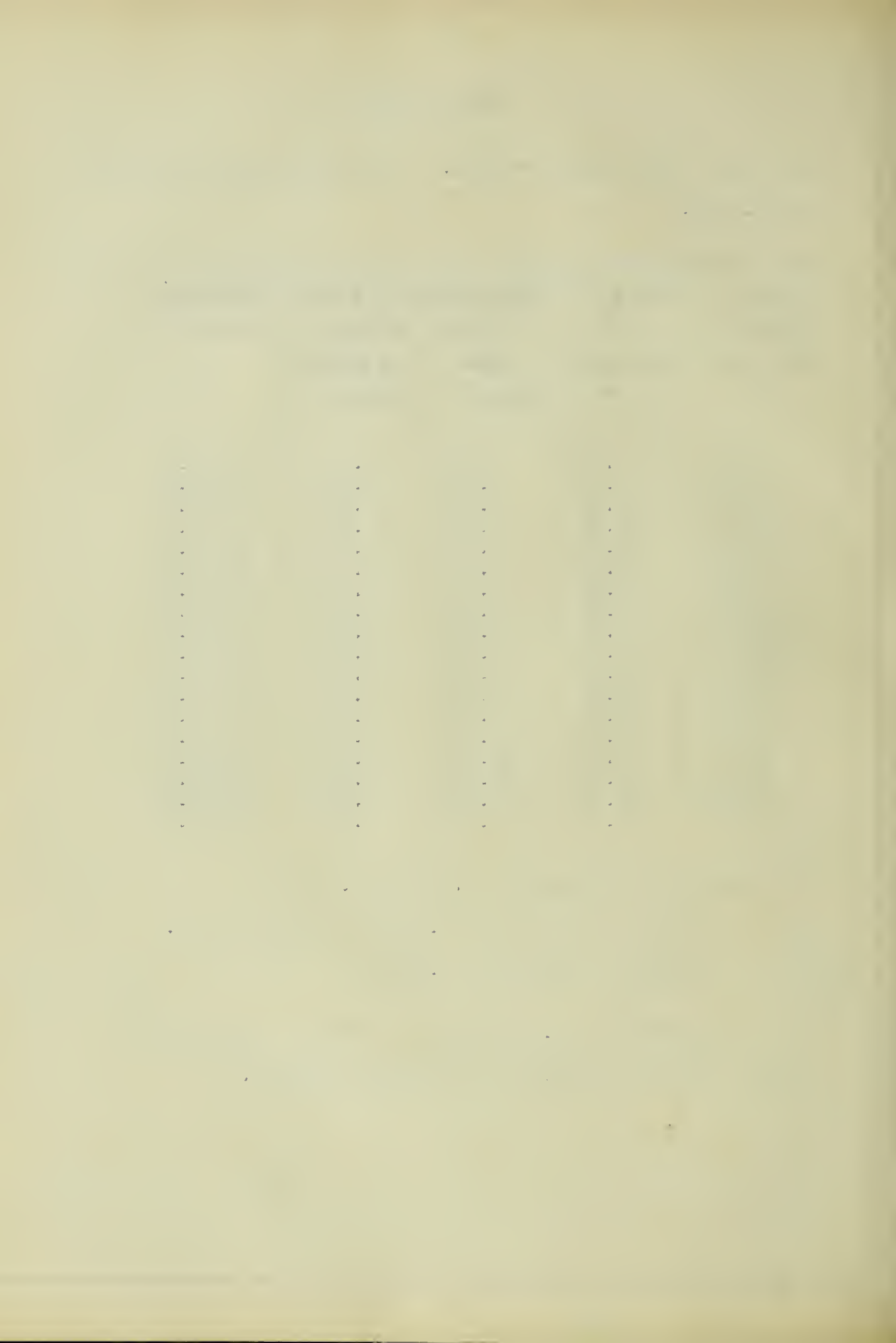
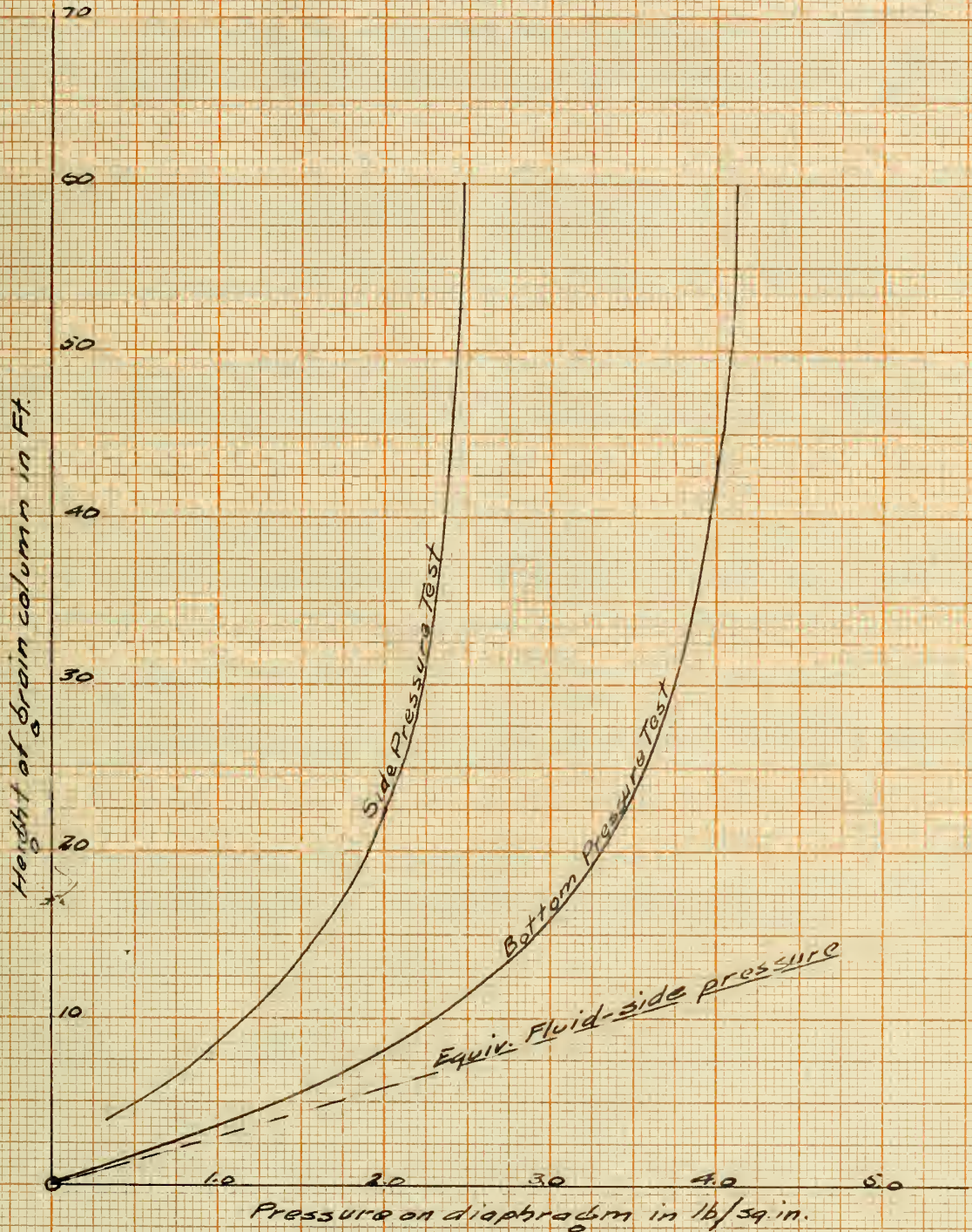


FIGURE 11 JAMIESON'S TESTS

Showing relation between pressure; both lateral and vertical, and height of grain in bin. (See Tables III and IV)



It is in this regard that a granular mass acts entirely different than does a true liquid. Some bins that were designed on the theory that grain acted as a fluid failed by crumpling of the walls due to this downward pressure caused by the weight of grain resting on them by friction. The depth at which these maxima are reached depends upon the coefficient of friction of the grain and wall and varies from 3.75 to 4.5 times the horizontal dimensions of the bin.

Third, the proportion of weight in the bin which is carried by the bottom and sides depends upon the following:

1. The angle of internal friction of the grain mass or the ratio of lateral to vertical pressure. For wheat the angle of repose equals 28 and, consequently, this ratio equals $\tan 28$ equals 0.6.

2. The coefficient of friction between the grain and the bin wall varies from 0.375 to 0.468, - mean value equals 0.41667.

3. The ratio of the area of the bin walls to the horizontal area, or the weight of the grain column set by the dimensions of the bin to be calculated.

4. The ratio of the diameter to the height of grain column.

With these factors determined the pressures in any bin can readily be calculated. His method of computation is by the "step process," to determine these pressures. An example will be worked out.

Take a bin 10 ft. square for wheat weighing 50#

per cubic foot. Area of cross section equals 100. For section 1 ft. deep, total area of walls equals 40. Starting with the top layer-- Total weight equals 5000#

(Due to the fact that the top layer gives such slight lateral pressure it may be neglected and simply considered as resting directly upon the second layer).

For second layer the part carried by the walls =

$$\frac{5000 \times 40}{100} \times 0.6 \times 0.41667 = 500\#$$

This 500# deducted from weight of second layer will leave 4500# for the vertical pressure of the second layer plus the weight of the first layer, which gives a total of 9500#.

The same method is continued until the desired depth is reached. Tables V and VI give the calculations for wheat bins 10 ft. and 20 ft. square respectively.

In comparing Table I with Table IV, Table I being Airy's experiments and Table IV those of Jamieson, the values for the lateral pressure are fairly consistent, but as to the pressure and the amount held up by the sides Airy's values are not even close to Jamieson's.

This difference from Jamieson's results is due to the fact that by Airy's calculations he finds that the maximum load on the bin bottom will be reached when the depth of grain in the bin reaches 3.5 times the horizontal dimensions of bin (This is seen by referring to Table I). As the depth of grain in the bin is increased above this point his table and formula gives a decreasing load on the bin bottom until

TABLE V.

Jamieson. Design by Step Process for Bin 10 ft. square,
wheat weighing 50# per cubic foot.

Height of Grain ft.	Weight in Bin lb.	Carried by sides per ft. lb.	Carried by sides Total lb.	Carried by Bottom per ft. lb.	Carried by Bottom Total lb.	Press. in lb/sq.in.	
						Lat.	Ver.
1	5000	000	000	4000	5000	0.00	0.35
2	10000	500	500	4500	9500	0.21	0.67
3	15000	950	1450	5050	13550		
4	20000	1355	2805	3645	17105		
5	25000	1730	4535	3280	20475	0.72	1.43
6	30000	2048	6573	2952	23427		
7	35000	2343	8916	2657	26084		
8	40000	2608	11525	2392	28475		
9	45000	2848	14373	2152	30627		
10	50000	3063	17436	1937	32564	1.28	2.28
15	75000	3856	35296	1144	39704	1.61	2.78
20	100000	4325	56080	675	43920	1.80	3.07
30	150000	4764	102120	236	47880	1.99	3.35
40	200000	4918	150740	82	49260	2.05	3.45
50	250000	4971	200258	29	49742	2.07	3.48
60	300000	4990	250090	10	49910	2.08	3.49
70	350000	4997	300032	3	59968	2.08	3.50
80	400000	4999	350012	1	49988	2.08	3.50

TABLE VI.

Jamieson. Design by Step Process for bin 20 ft. square and
80 ft. deep. Wheat weighing 50# per cubic foot.

Height of Grain Ft.	Weight in Bin lb.	Carried by Sides per ft. lb.	Carried by Sides Total lb.	Carried on Bottom per ft lb.	Carried on Bottom Total lb.	Press. in lb/sq.in.	
						Lat.	Ver.
1	20000	000	000	20000	20000	0.00	0.34
2	40000	1000	1000	19000	39000	0.21	0.66
3	60000	1950	2950	18050	59050		
4	80000	2853	5803	17147	74197		
5	100000	3710	9513	16290	90487	0.77	1.54
6	120000	4524	14037	15476	105963		
7	140000	5298	19335	14702	120665		
8	160000	6033	25368	13967	134632		
9	180000	6732	32100	13268	147900		
10	200000	7395	39495	12605	160505	1.54	3.73
15	300000	10347	85317	9753	214683	2.14	3.65
20	400000	12453	143395	7547	256605	2.59	4.36
25	500000	14160	210955	5840	289045	2.95	4.91
30	600000	15481	285995	4519	314145	3.23	5.34
35	700000	16504	366434	3496	333565	3.44	5.67
40	800000	17295	451405	2705	348595	3.60	5.93
45	900000	17907	539729	2093	360321	3.73	6.12
50	1000000	18380	630730	1620	369220	3.83	6.28
60	1200000	19030	818430	970	381570	3.97	6.49
70	1400000	19310	1011041	581	388959	4.05	6.61
80	1600000	19652	1206612	348	393388	4.09	6.67

the grain reaches a depth where the only pressure on the bottom is due to a pyramid or cone of grain whose sides are at the greatest angle of repose of the grain, overlooking the fact that the weight carried by the bottom, having once attained a maximum cannot be decreased by any increase in lateral pressure. This change in bottom pressure is illustrated by Plate III. Due to this fact Airy's calculations should be given very little weight.

As regards Jansen's tests as compared with Jamieson's, they are almost identically the same. This fact is illustrated by the two sets of results shown in Table 7. It will be noted that Jansen's test bin was 11.8 in. square while Jamieson's was 12 in. square.

CONCLUSION.

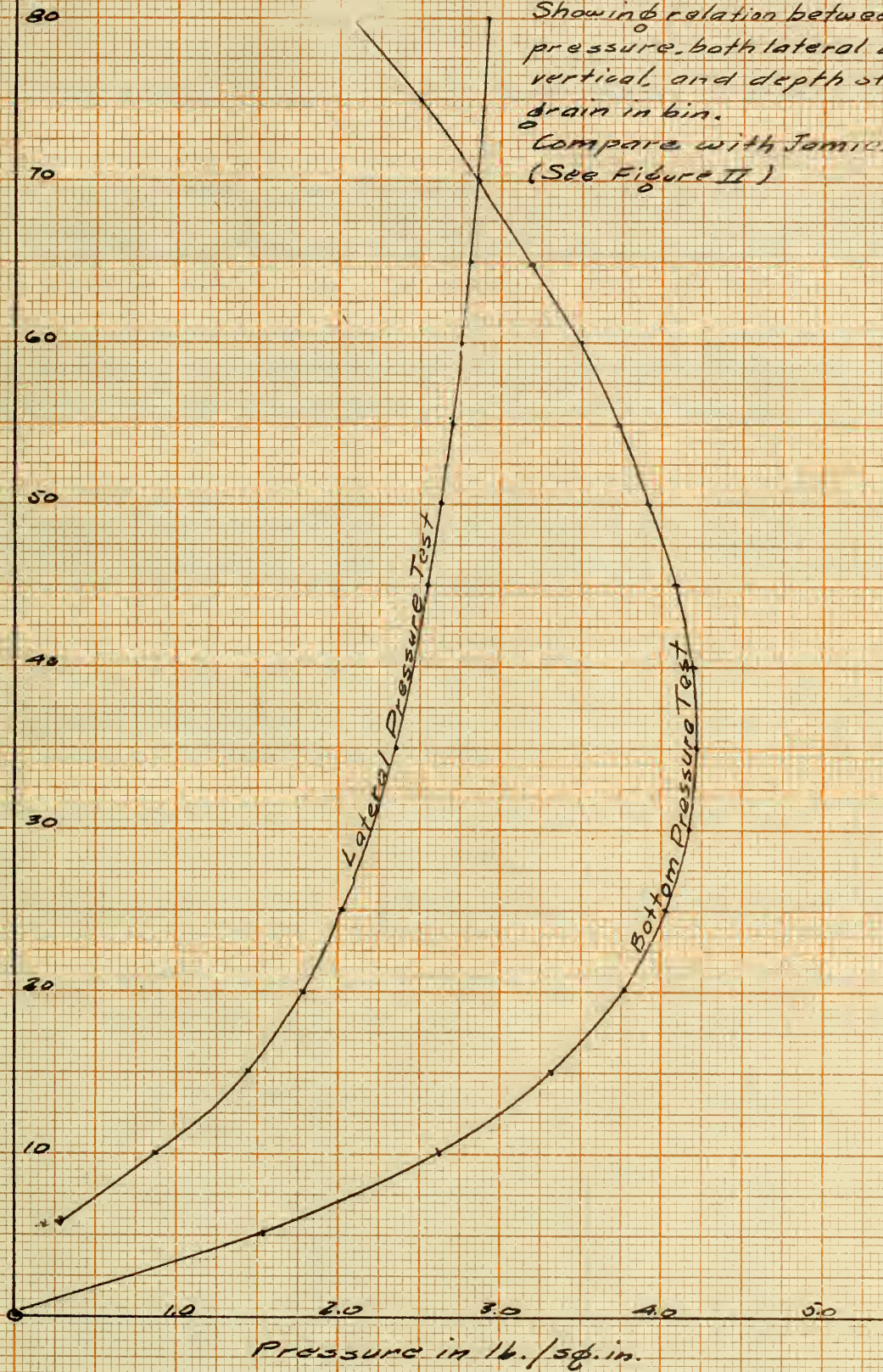
From the data presented it is concluded that Jamieson's results are to be depended upon as they are substantiated by other experimentors. His conclusions and method of design are all the more valuable because they are founded directly upon experiment and hence, bins designed by his method should be both economical and scientific.

These sets of experiments establish, without shadow of doubt, that there is no analogy between the lateral pressure as exerted in a granular mass and that exerted by a true liquid of the same specific gravity. The divergence is illustrated by referring to Figure 2.

FIGURE III AIRY'S TESTS

Showing relation between
pressure, both lateral and
vertical, and depth of
grain in bin.
Compare with Jamieson's
(See Figure II)

Depth of grain in bin - Ft.



COMPARISON OF JAMIESON'S RESULTS WITH THOSE OF JANSEN

--TABLE VII

Size of bin 12 inches square. Wheat used weighed 50# per cubic ft
Jamieson's

Grain Weighed into Bin lb.	Ht. of Grain Col. in.	Equiv. Fluid Press. in. of water	Press. of Grain on Diaph. in. Water	Grain Carried on Bottom		Grain Carried on Sides	
				Weight lb.	% Tot. Weight of Grain	Weight lb.	% Total Weight of Grain.
25	6	4.81	3.75	19.483	77.93	5.517	22.07
50	12	9.62	5.87	30.524	61.04	19.476	38.96
75	18	14.43	7.00	36.368	48.49	38.632	51.51
100	24	19.24	7.87	40.914	40.91	59.086	59.09
125	30	24.05	8.25	42.863	34.29	82.137	65.81
150	36	28.86	8.63	44.811	29.87	105.189	71.13
175	42	33.67	8.75	46.110	26.35	128.890	73.65
200	48	38.48	9.00	46.759	23.37	153.241	76.63
225	54	43.29	9.44	49.032	21.79	175.968	78.21
250	60	48.10	9.56	49.682	19.87	200.318	80.13
275	66	52.81	9.75	50.656	18.42	224.344	81.58
300	72	57.62	9.75	50.656	16.88	249.344	83.12
325	78	62.53	9.75	50.656	15.58	274.344	84.42

Carried on bottom = 50.656#

" " side = 274.344#

TABLE VIII

Size of bin, 11.8 in. x 11.8. Wheat weighed 50# per cubic foot.
Jansen's--

Grain Weighed into Bin lb.	Height of Grain Col. in.	Press. of Grain on Bottom of Bin in. Water	Grain Carried on Bottom		Grain Carried on Sides	
			Weight lb.	% Total Weight of Grain	Weight lb.	% Total Weight of Grain.
23.2	6.0	3.75	81.20	18.84	4.36	18.80
46.4	12.0	5.75	62.26	28.89	17.51	37.74
69.6	18.0	7.19	51.88	36.11	33.49	48.62
92.8	24.0	8.13	43.98	40.82	51.98	56.02
116.0	30.0	8.75	37.90	43.96	72.04	62.10
139.2	36.0	9.19	33.16	46.16	93.04	66.84
162.4	42.0	9.50	29.39	47.73	114.67	70.61
185.6	48.0	9.62	26.05	48.35	137.25	73.95
198.4	51.3	9.62	24.36	38.35	150.06	75.64
208.8	54.0	9.62	23.15	48.35	160.45	76.85
301.6	78.0	9.87	16.44	49.61	251.99	83.56

-EARTH PRESSURE

The subject of lateral pressure as exerted in a mass of earth is one that has been the cause of a great amount of discussion for the last thirty or forty years, with the result that paper after paper has been written all of which accomplish the conclusion that experiment and not theory must decide what this pressure is in amount. The various formulas derived from purely theoretical treatments of the question are of value to the extent that they furnish a means of fairly exact approximation in the design of retaining walls; but to place the subject upon a real scientific basis there must be greater harmony between fact and theory.

If earth were a homogeneous mass and devoid of cohesive action, these different deductions would be more nearly correct; but this is not so and the assumptions being incorrect precludes exactness in the final results. With the case of grain these assumptions would be correct so long as the grain was in a dry state, and the formulas as given for the determination of lateral pressure in earth would in very little error when applied to grains.

Due to the fact that earth is such an inconsistent material some assumptions necessarily have to be made in order to arrive at any deductions, but more experimental data is required than is now at hand to determine just what assumptions cannot be made.

Several different papers on the subject will be taken up and a synopsis given of the methods employed to determine this lateral pressure.

Benj. Baker's work upon the determination of lateral pressure in earth consists of a series of tests carried on, indirectly and directly, to verify the theories that existed at that time. He advances a method for getting the relation between the height of a retaining wall and the thickness at the toe, which method is founded on several different theories. Figure 5 illustrates his method. In the diagram shown, $abcd$ is a square, ab a triangle with a ratio of sides of 1: 3 and ahd a parabolic curve of the equation $X = 100Y$. Take for example a slope of repose of 2:1, draw cf at this slope and bisect the angle made by it and the back face of the wall (represented by ac). The length of eh in part of total side bd gives the equivalent fluid pressure in terms of a cubic foot of earth; and the line, ei gives the thickness of the wall in terms of the height. Thus for this case, assuming the wall and backing to weigh 100# per cubic foot, the pressure acting on the wall would be that due to a fluid weighing 38# per cubic foot, and the thickness of the wall would be 35.7% of the height. If the masonry were heavier than the backing, the required thickness of wall would be reduced in inverse proportion to the square root of the respective heights. The accompanying table gives the equivalent fluid pressure and thickness of wall for material of equal weight for different angles of repose. (See table 8)

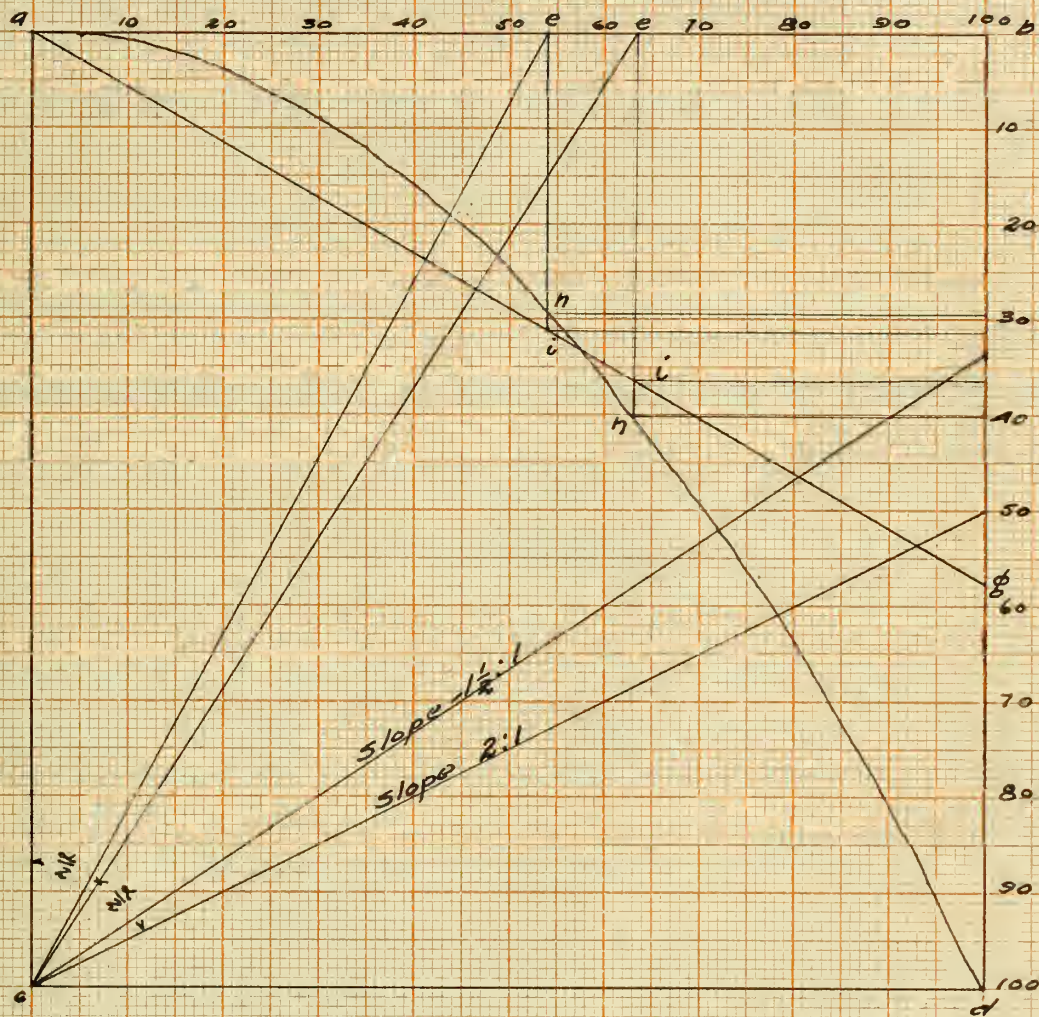


FIGURE 5

BENJ. BAKER'S METHOD

For materials of equal weight
(wall and filling)

Equation of parabolic curve - $x^2 = 100y$.



TABLE VIII

Ratio of Hor to Ver	.5	.6	.7	.8	.9	1.0	1.1	1.2
Fluid Press #	5.6	7.7	10.0	13.4	14.8	17.2	19.6	22.0
Thickness in parts of height	.136	.160	.182	.203	.222	.239	.256	.270

1.3	1.4	1.5	1.6	1.7	1.8	2.0	3.0	4.0	8.0
24.3	26.5	28.7	30.7	32.8	34.6	38.2	52.0	61.0	100.0
.284	.297	.31	.32	.33	.34	.357	.416	.451	.578

If all problems in retaining walls could be solved thusly it would certainly be a simple matter, but it is not so as his assumptions are faulty. This method will, however, furnish a means of approximation as to the action of the earth when held by a retaining wall.

He concludes, that, as a result of his own experience, the thickness of retaining walls in ground of average character should be $1/3$ of the height from top of footings. Also that if any material is taken out to form a face panel, $3/4$ of it should be put back in the form of pilasters.

The paper that seems to be of greatest value is that of Cains published in 1888. His deductions are founded upon

principles by Wayranch, Columb and Rankine. He takes into account the friction between the earth and the back of the wall which causes the lateral thrust to make an angle with the normal to the wall, equal to the angle of friction between the two materials. An outline of his method will be given:-

The Theory of Retaining Walls as given by Cain, (1888)

The earth is considered as a homogeneous, compressible mass, made up of particles possessing the resistance to sliding over each other called friction, but without cohesion. The graphical method will be outlined as this method is simpler than the analytic since the latter contains such a maze of formulas each containing so many different factors that substituting in them would be a big task. It is assumed, to begin with, that the earth behind the wall, whether the top surface is a plane or not, has a tendency to slide along some such plane surface of rupture as $A_1, A_2, \dots$. The principle of mechanics relative to the stability of a granular mass is that as first stated by Rankine, and is as follows: "It is necessary to the stability of a granular mass, that the direction of the pressure between the portions into which it is divided by any plane should not, at any point, make with the normal to that plane an angle exceeding the angle of repose." Referring to Fig. VI let $\overline{ABDC}$ be the retaining wall, $A_0, A_1, A_2, \dots$ the possible planes of rupture. $\overline{AF}$ is the natural slope of the earth.

ϕ = angle of slope of earth.

ω = angle made by plane of rupture with perp. from A

ϕ' = " of friction between earth and wall back.

G = weight of prism of rupture.

S = resultant force acting on plane of rupture.

E = lateral thrust against wall.

W = weight of wall. R = resultant acting on foundation.

Consider, for example, that the real plane of rupture is $\overline{AI}$. Draw G from the c.of g. of prism and equal to the weight of this prism 1' thick. Draw $\overline{S}$ making the angle ϕ with a perpendicular to plane of rupture $\overline{AI}$ and E making the angle ϕ' with the normal to the back face of the wall, $\overline{AC}$. Complete the parallelogram of forces and scale E and S. Now from the c.of g. of the wall draw $\overline{W}$. $\overline{E}$ is known in direction and quantity, whence the resultant $\overline{R}$ can be found by completing the rectangle. The point of application of $\overline{E}$ is taken as $1/3 \overline{AC}$ for all cases of horizontal fill and $0.4 \overline{AC}$ for surcharge.

The plane of rupture is found by the following method: (see figure 7). Draw the possible planes of rupture as $\overline{A1}$, $\overline{A2}$, $\overline{A3}$,..... The triangles $Ao1$, $Ao2$, $Ao3$ are now reduced to similar triangles having the same base as $\overline{A2}$. Draw the parallels $\overline{O0'}$, $\overline{11'}$, $\overline{33'}$, to line $\overline{A2}$ to intersect with a perpendicular to $\overline{A2}$, passing through the point $\overline{2}$. Then the triangle $Ao2$ is equivalent to the triangle $Ao'2$. Also $A26$ is equivalent to $A26'$. Thus the area $\overline{A026A}$ is replaced by $\overline{A0'26'A}$; and the weights of the corresponding prisms (e = weight per cu. ft. of earth) is

$1/3 \overline{A2} \times \overline{O'6'} \times e$; so that if we use $\overline{O'1'}$, $\overline{O'2'}$, $\overline{O'3'}$
 to represent the weights of the successive prisms $\overline{AO1}$, $\overline{AO2}$,
 $\overline{AO3}$, on the force diagram given below we have simply
 to multiply the value of E , given by construction, by $1/3 e \cdot \overline{A2}$
 to give its true value in pounds. Now lay off the successive
 values of $(\phi + \omega)$, measured from horizontal $\overline{OI}$. $a_1, a_2, a_3, \dots$
 are the points of intersection between the different possible
 planes of rupture with the arc described by a radius $= \overline{A\phi}$.
 Then from ϕ draw the arc $bs_1, \dots$ equal in radius to $\overline{A\phi}$.
 Make $\overline{bs_1} = \overline{tc_1}$, $\overline{ss_1} = \phi a_1$, $\overline{ss_2} = \phi a_2, \dots$. Draw the lines ϕs_1 ,
 $\phi s_2, \dots$, etc. Then the angle $\angle b\phi s_1 = (\phi + \omega_1)$.
 Lay off on the vertical $\overline{A\phi}$ the distances $\overline{gg_1}, \overline{gg_2}, \dots$
 equal to $\overline{O'1'}$, $\overline{O'2'}$, $\overline{O'3'}$ and draw through these points
 parallels to direction of E to intersect with the lines ϕs_1 ,
 $\phi s_2, \phi s_3, \dots$ which intersections call c_1, c_2, c_3, c_4 re-
 spectively. Connect them by the curve as shown and the
 greatest line corresponds to the real plane of rupture. In
 the construction shown $\overline{\phi a_4 c_4}$ is the greatest, hence $\overline{A4}$ is the
 plane of rupture.

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Am. Rail. Eng. and Maintenance of way Assoc. Feb, 1909.

This method is based on Rankine's formulas which considers that
 the earth is a granular mass with some assumed or determined
 slope or angle of repose. In applying this method it is
 immaterial whether the force representing the earth pressure
 is considered as acting directly on the back of the wall or
 if it is considered as acting upon a vertical plane passing

through the extreme back of the footing. In the latter case the force representing the lateral earth pressure must be combined with (1) the vertical force representing the weight of the earth prism between the back of the wall and the vertical plane considered, and (2) combined with the vertical force representing the weight of the wall itself.

Assuming the repose of the earth to $1\frac{1}{2}:1$ the force per linear foot of wall, representing the resultant of the earth pressure back of the wall acting on the vertical plane referred to, will be as shown in Fig. 8, for the case of horizontal fill behind the wall. Fig. 9 shows the results when there is surcharge having a slope of $1\frac{1}{2}:1$

y = weight of a cubic ft. of earth

h = height of wall from bottom of footing to top.

In case the wall is to support a track load in addition to the earth pressure then the lateral thrust, by the same method will be as shown in Fig. 10.

REFER TO
FIGURE-9

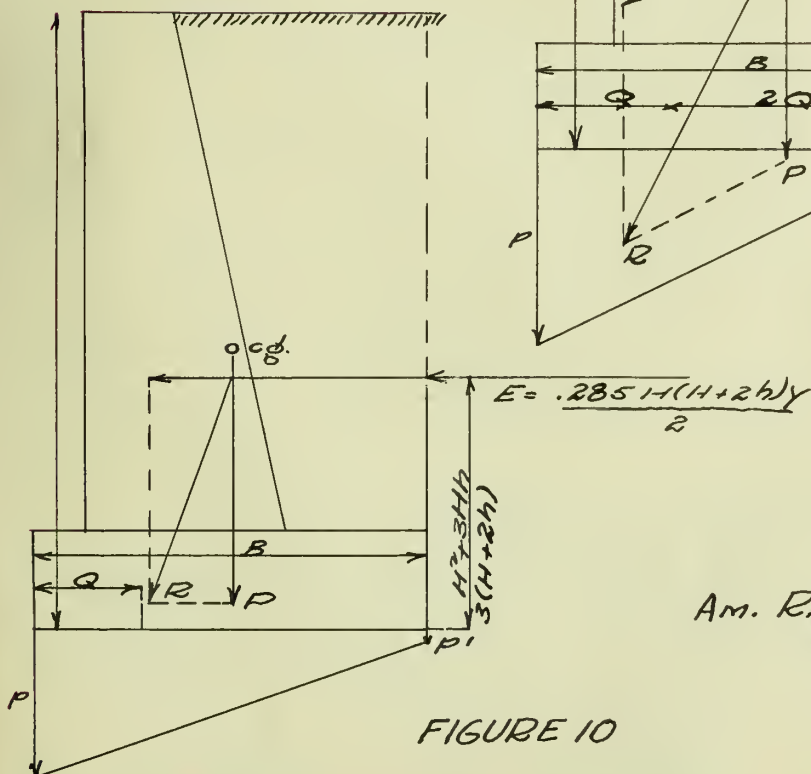
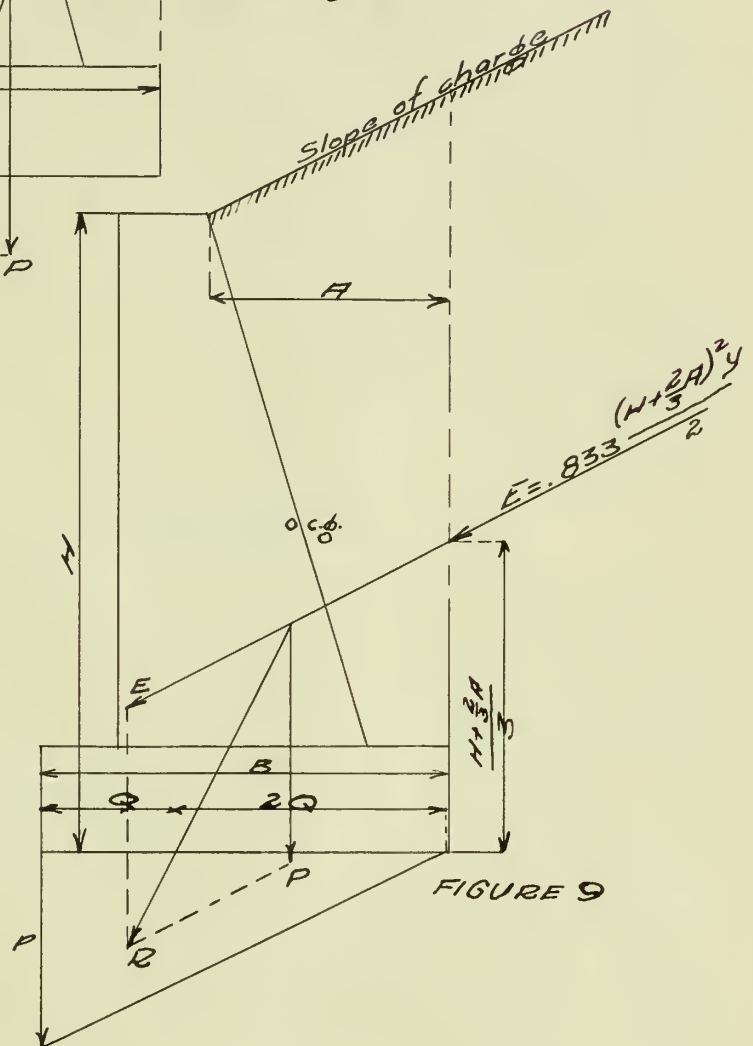
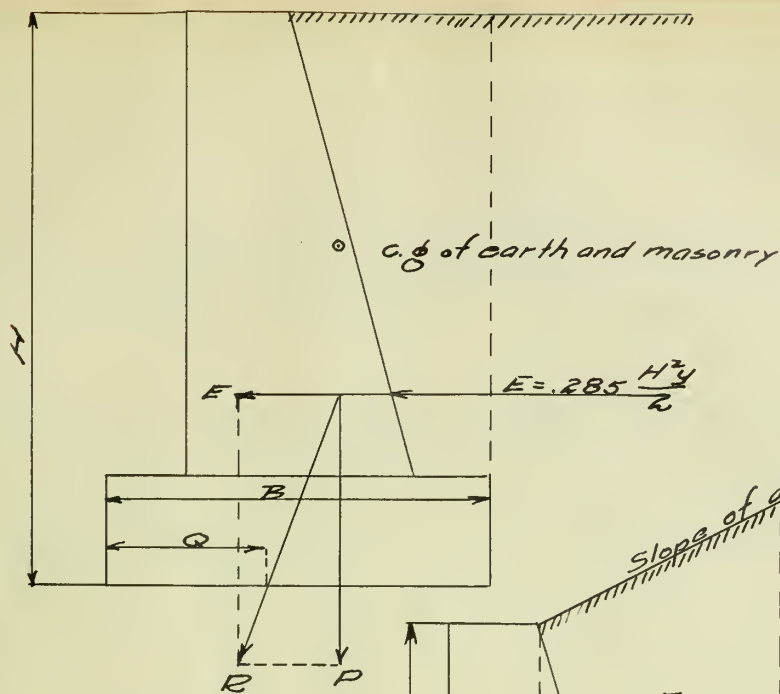
(When Q is equal to
or greater than $\frac{B}{3}$:-

Pressure at toe = $P =$
 $(4B - 6Q) \frac{P}{B^2}$

Pressure at heel = $P' =$
 $(6Q - 2B) \frac{P}{B^2}$

When Q is less than $\frac{B}{3}$:-

$$P' = \frac{2P}{3Q}$$



METHOD

AS GIVEN

BY

AM. RAILWAY AND MAINTENANCE °. WAY
ASSOCIATION

FEBRUARY - 1909

Figure 11 shows a type of coal bin which is very common and the analysis of pressures for this special case will be given.

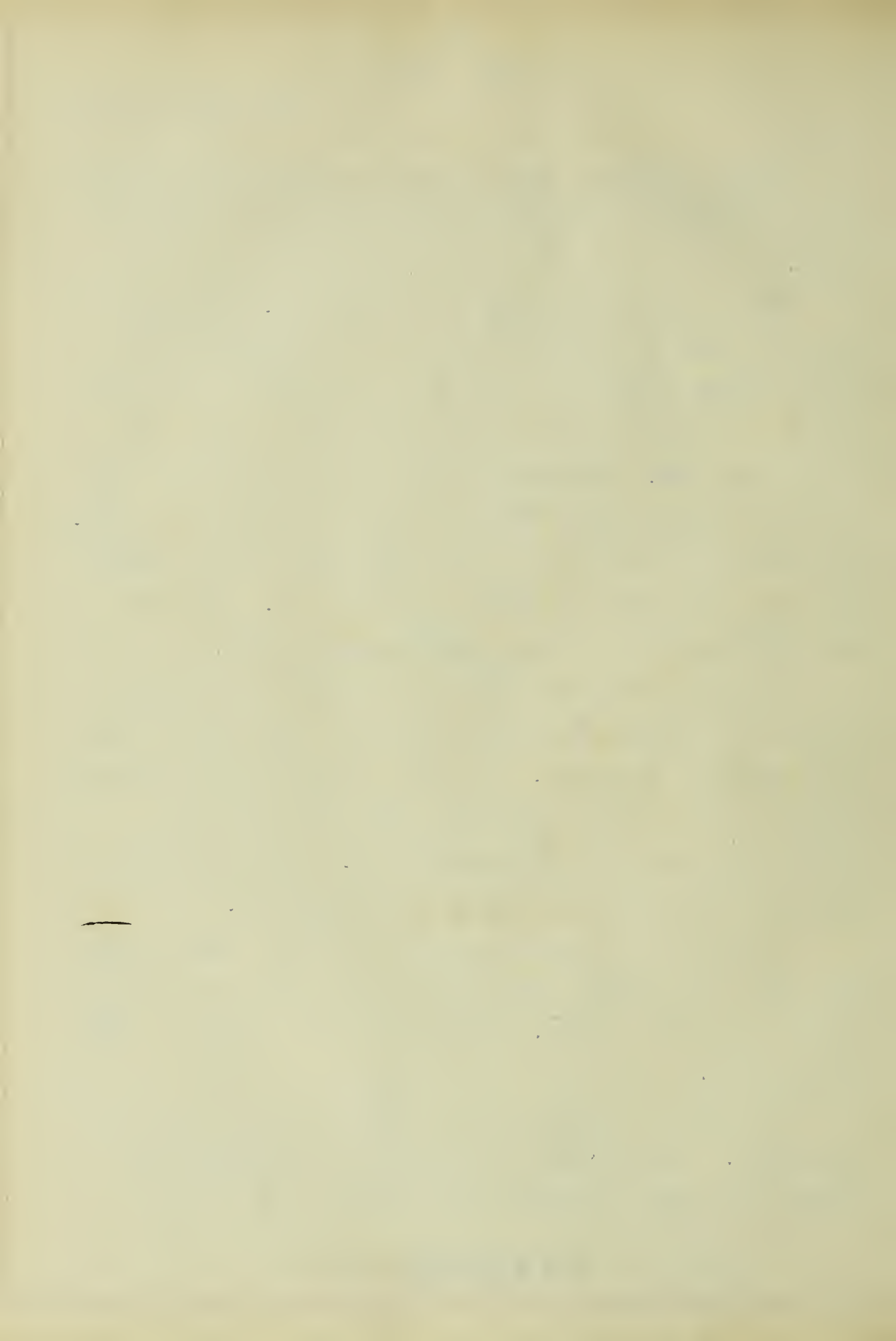
$\overline{AC}$ is the natural slope of the material. $\overline{AP}$ represents the plane of rupture of the coal which will give the maximum thrust against the side $\overline{AM}$ and to fulfill this condition it bisects the angle between the natural slope $\overline{AP}$ and the wall $\overline{AM}$. Suppose the bin to be level full to $\overline{MN}$, then the triangular mass $\overline{MOA}$, will exert a pressure upon $\overline{MA}$. Now suppose the bin to be full up to $\overline{XY}$, then the mass $\overline{MOA}$ will carry the weight of $\overline{MXPO}$ directly on it. Thus the mass $\overline{AMXP}$ produces pressure against the side $\overline{AM}$.

Supposing this mass were free to slide down the plane of maximum pressure $\overline{AP}$, without friction; the weight is the force causing it to slide. Let $\overline{ab}$ represent this force and resolve it into the two components, $\overline{ac}$ and $\overline{ad}$, horizontal and parallel to $\overline{AP}$ respectively. Now $\overline{ap}$ will represent the reaction of $\overline{ac}$, equal and opposite.

In addition to this the weight of the mass $\overline{AMXP}$ rests upon $\overline{AM}$ and represent this weight by $\overline{st}$ which force is resolved into $\overline{sz}$ and $\overline{sf}$, perpendicular and parallel to $\overline{AM}$ respectively.

The points of application of these forces will now be determined. The material is treated as a liquid and the point x below $\overline{xy}$ is found by the equation

$$d = \frac{2}{3} \frac{H^3 - h^3}{H^2 - h^2}$$



ENG. NEWS METHOD OF DESIGN
OF
COAL-BIN SHOWN

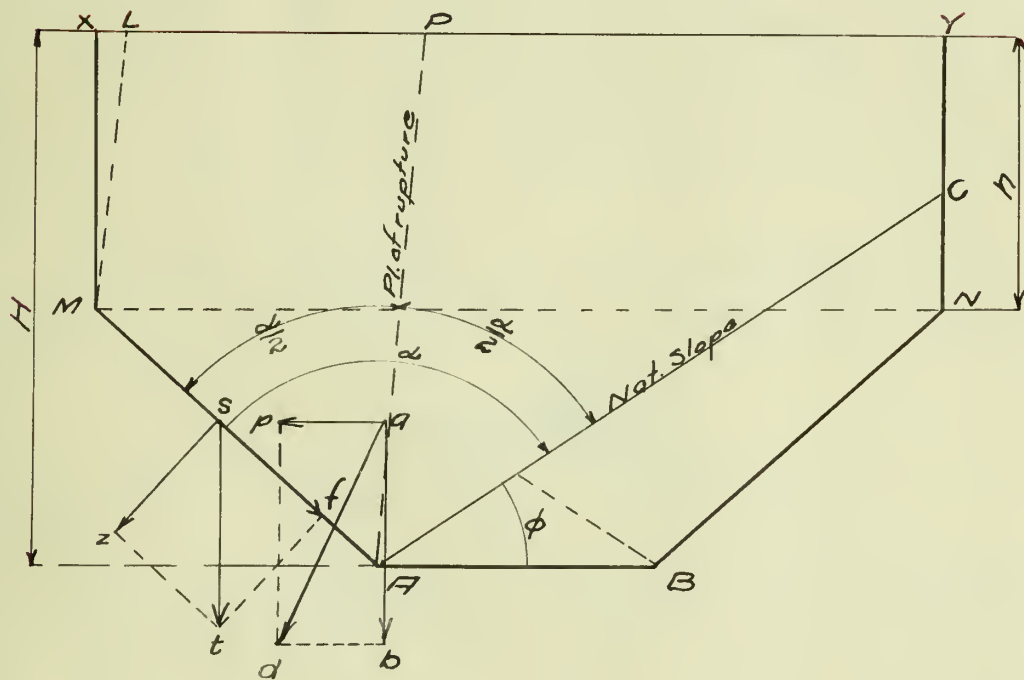


FIGURE 11

The forces $\overline{ap}$ and $\overline{st}$ are considered concentrated, thus the point $\overline{a}$ is determined.

The forces acting on $\overline{AM}$ and their point of application is now determined and it is a very simple matter to compute the proportion of this resultant force which is transferred to $\overline{M}$.

The pressure against the side $\overline{MX}$ is found by the same general method. $\overline{ML}$ is the plane of rupture and the horizontal thrust is due to the wedge-shaped mass $\overline{XML}$ sliding down the plane $\overline{ML}$. This force is considered as acting $1/3 \overline{h}$ up from $\overline{M}$.

If the bin were heaped full the process would be the same only it will be seen that the horizontal thrust is directly increased.

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R. W. Dull Formulas & Tables for Bin Design.

The following tables and formulas are for the design of bins and hoppers for the following material; bituminous coal, anthracite coal, sand and ashes. The different cases are taken up so that all styles of bins are found herewith. The formulas are based upon Cain's theory as taken up under the subject of retaining walls.

Nomenclature

ϕ = angle of repose, measured from horizontal.

ϕ' = " of friction between material and bin wall =
angle between thrust and normal to plate.

P = total thrust against his wall for a section 1' thick.

N = normal comp. of P .

ϵ = slope of surface of materials measured from horiz.

∞ = angle which bin wall makes with vertical.

ω = weight of material per cubic foot.

W = total weight for section 1' thick.

Case I.

Vertical wall. Surface level.

$$P = \left(\frac{\cos \phi}{n+1} \right)^2 \frac{\omega h^2}{2 \cos \phi'} \quad N = \left(\frac{\cos \phi}{n+1} \right) \frac{\omega h^2}{2}$$

where

$$n = \sqrt{\frac{\sin(\phi + \phi') \sin \phi}{\cos \phi'}} \quad \text{If } \phi = \phi' \text{ then } n = \sqrt{2 \sin \phi}$$

$$\text{and } P = \frac{\omega h^2}{2} \frac{\cos \phi}{(1 + 2 \sin \phi)^2} \quad N = \frac{\omega h^2}{2} \frac{\cos^2 \phi}{(1 + 2 \sin \phi)^2}$$

If $\phi' = 0$ which corresponds to a very smooth wall,

$$P = N = \frac{\omega h^2}{2} \tan^2 \left(45^\circ - \frac{\phi}{2} \right)$$

Table of constants for Steel Plate Bin.

<u>Material</u>	<u>ϕ</u>	<u>ϕ'</u>	<u>w</u>	<u>P</u>	<u>N</u>
Bitum. Coal	35	18	50#	6.13 h	5.83 h
Anthracite Coal	27	16	52	8.73 h	8.39 h
Sand	34	18	90	11.50 h	10.93 h
Ashes	40	31	40	4.02 h	3.44 h

Case II

Vertical wall. Surcharged surface.

$$P = \left(\frac{\cos \phi}{n+1} \right)^2 \frac{\omega h^2}{2 \cos \phi'} \quad N = \left(\frac{\cos \phi}{n+1} \right)^2 \frac{\omega h^2}{2}$$

$$\text{where } n = \sqrt{\frac{\sin(\phi + \phi') \sin(\phi - \epsilon)}{\cos \phi \cos \epsilon}}$$

$$\text{If } \epsilon = \phi \quad P = \frac{\cos^2 \phi}{\cos \phi'} \frac{\omega h^2}{2} \quad N = \cos^2 \phi \frac{\omega h^2}{2}$$

$$\text{If } \epsilon = \phi = \phi'$$

$$P = N = \frac{\cos \phi \omega h^2}{2}$$

Note: Direction of P in this case is parallel to surface slope.

Table of constants for steel plate bins.

	ϕ	ϕ'	$\frac{w}{50\#}$	$\frac{P}{h}$	$\frac{N}{h}$
Bit. Coal	35	18	50#	17.65 h	16.75 h
Anth. "	27	16	52	21.45 h	20.60 h
Sand	34	18	90	32.50 h	30.90 h
Ashes	40	31	40	13.70 h	11.73 h

Case III

Wall leaning outward, but within "limiting plane"* surface level.

$$P = \left[\frac{\cos(\phi - \alpha)}{(n+1)\cos\alpha} \right]^2 \frac{\omega h^2}{2 \cos(\phi' + \alpha)}, \quad N = \left[\frac{\cos(\phi - \alpha)}{(n+1)\cos\alpha} \right]^2 \frac{\omega \cos \phi' h^2}{2 \cos(\phi' + \alpha)}$$

where
$$n = \sqrt{\frac{\sin(\phi + \phi') \sin \phi}{\cos(\phi' + \alpha) \cos \alpha}}$$

Case IV

Same conditions as Case III; only surcharged surface.

$$P = \left[\frac{\cos(\phi - \alpha)}{(n+1)\cos\alpha} \right]^2 \frac{\omega h^2}{2 \cos(\phi' + \alpha)}, \quad N = \left[\frac{\cos(\phi - \alpha)}{(n+1)\cos\alpha} \right]^2 \frac{\omega h^2 \cos \phi'}{2 \cos(\phi' + \alpha)}$$

where
$$n = \sqrt{\frac{\sin(\phi + \phi') \sin(\phi - \epsilon)}{\cos(\phi' + \alpha) \cos(\alpha - \epsilon)}}$$

* Limiting plane is the plane of rupture.

Case V

Wall sloping outward beyond limiting plane. Level surface.

$$P = \tan^2(45^\circ - \frac{\phi}{2}) \frac{wh^2}{2}$$

$$W = \text{weight of } \triangle ABC = \frac{w \tan \alpha h^2}{2}$$

$$E = \sqrt{W^2 + P^2} = \frac{wh^2}{2} \sqrt{\tan^2 \alpha + \tan^4(45^\circ - \frac{\phi}{2})}$$

$$\tan \delta = \frac{\tan \alpha}{\tan^2(45^\circ - \frac{\phi}{2})}$$

$$Q = E \cos(\delta - \alpha)$$

$$T = E \sin(\delta - \alpha)$$

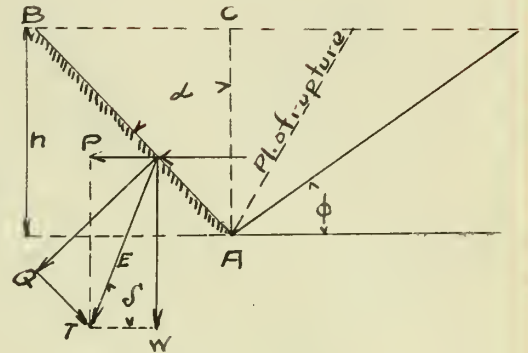


Table of constants for Steel Plate Bins

	ϕ	α	$\frac{W}{h}$	$\frac{E}{h}$	Q	T	δ
Anth. Coal	27	45	52#	27.8 h	25.3 h	11.5 h	69°-26'
Sand	34	45	90	46.8 h	40.8 h	22.87 h	74°-16'
Ashes	40	45	40	20.46 h	17.2 h	11.07 h	77°-46'
Bit. Coal	35	45	50	25.88 h	22.4 h	12.9 h	74°-54'

Case VI

Same as Case V with surcharge:-

$$P = \cos \epsilon \frac{\cos \epsilon - \sqrt{\cos \epsilon - \cos \phi}}{\cos \epsilon + \sqrt{\cos^2 \epsilon - \cos^2 \phi}} \frac{wh^2}{2}$$

(Direction of P is parallel to surface of surcharge)

$$W = \frac{w \sin \alpha \cos(\epsilon - \alpha)}{2 \cos^2 \alpha \cos \epsilon} h^2, \quad E = \sqrt{P^2 + W^2 + 2PW \sin \epsilon}$$

If $\epsilon = \phi$

$$E = \frac{w \cos(\phi - \alpha)}{2 \cos^2 \alpha \cos \phi} h^2$$

Table of constants for Steel Plate Bins.

Bit, Coal	ϕ 35	ϵ 35	α 45	$\frac{w}{50\#}$	$\frac{E}{90.34}$ h
Ant. "	27	27	"	52	78.7 h
Sand	34	34	"	90	159.0 h
Ashes	40	40	"	40	80.6 h

The tables of pressure for vertical wall bin is given for depth of from 1' to 30' for the foregoing materials.

The tables for bituminous coal and sand will be given:-

Table 9

Total pressure for depth "h" bituminous coal--

$$w = 50\#$$

$$\phi = 35^\circ$$

Depth h	N Surface- level- $\phi' = 18^\circ$	N Surface level $\phi' = 0$	N Surcharge = ang. of repose $\phi = \epsilon$ $\phi' = 18^\circ$	N Surcharge = ang. of repose $\epsilon = \phi$ $\phi' = 0$
1	5.83	6.75	16.75	20.5
2	23.32	27.00	67.0	82.0
3	52.47	60.75	150.75	184.5
4	93.4	108.00	268.0	328.0
5	145.7	168.75	418.75	512.
6	209.4	243.0	603.	738
7	208.0	333.0	821.	1065.
8	273.0	452	1072	1312
9	472	547	1357	1661
10	583	675	1675	2050
11	705	817	2027	2481
12	840	972	2412	2952
13	985	1141	2831	3465
14	1143	1323	3283	4018
15	1312	1519	3769	4613
20	2332	2700	6700	8200
25	3644	3888	10469	12813
30	5247	6075	15075	18450

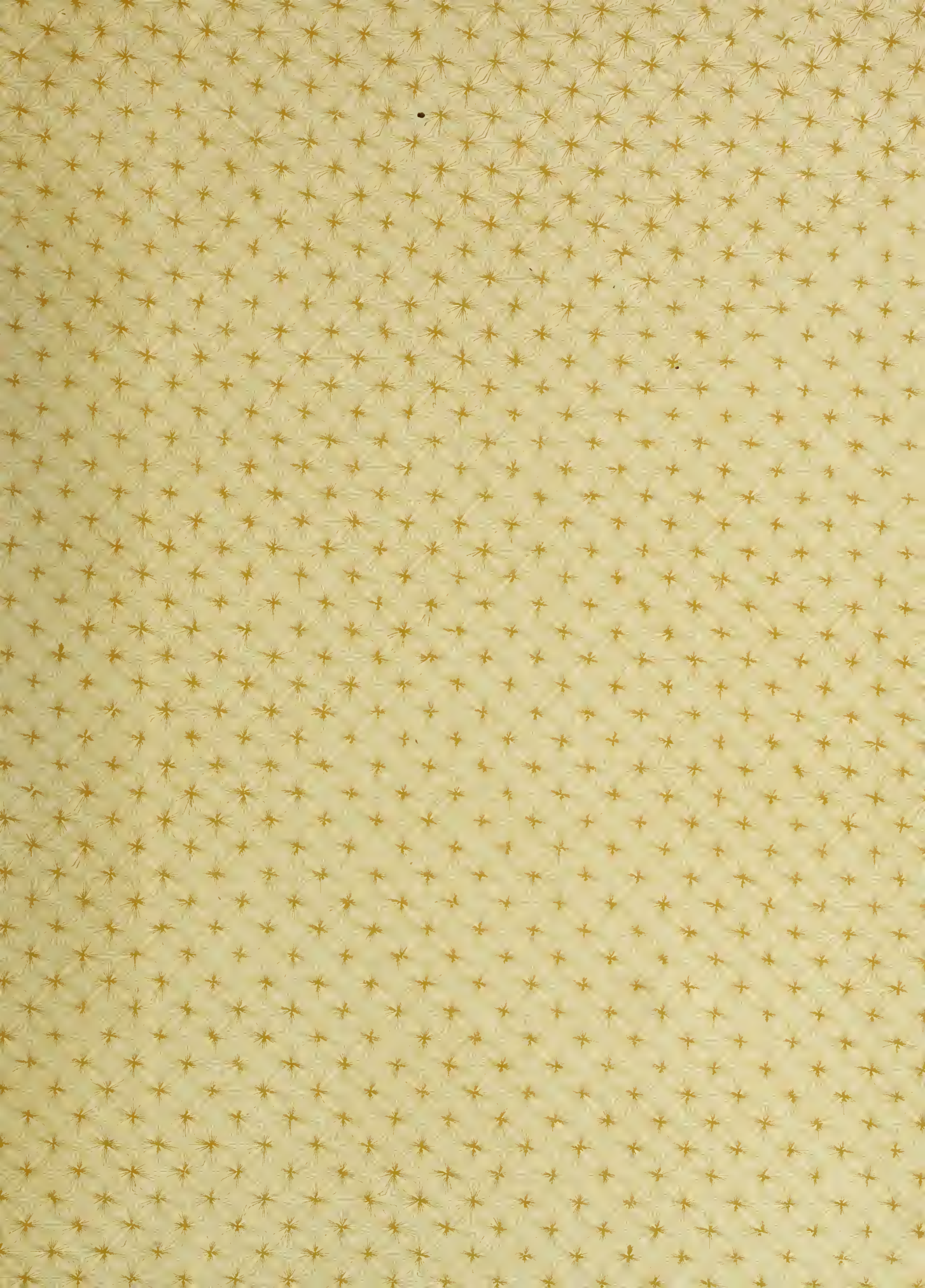
Table 10-- Total Pressure for depth "h" of sand.

$w = 90\#$ per cubic foot. $\phi = 34^\circ$

Depth	N	N	N	N
"h"	Surface- level	Surface- level	Surcharge= ang. rep.	Surcharge= ang. rep.
	$\phi' = 18^\circ$	$\phi' = 0$	$\epsilon = \phi$ $\phi' = 18^\circ$	$\epsilon = \phi$ $\phi' = 0$
1	10.93	12.72	30.9	37.3
2	43.7	50.8	123.6	143.0
3	98.5	114.5	278.0	336.0
4	175.0	203.7	494.0	596.
5	273	318.0	772.0	933
10	1093	1272	3090	3730
15	2460	2862	6960	8390
16	2798	3256	7910	9549
17	3159	3676	8930	10780
18	3541	4121	10012	12095
19	3946	4592	11155	13465
20	4372	5088	12360	14920
25	6831	7950	19313	23313
26	7387	8599	20889	25215
27	7968	9273	22536	27192
28	8569	9972	24235	29243
30	9837	11488	27810	33570

Sides are vertical for Tables I and II.





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